of these expressions lies in their mathematical simplicity, which should facilitate further analytical studies.

References

[1] Spreiter, J. R. and Sacks, A. H., "The Rolling Up of the Trailing Vortex Sheet and Its Effect on the Downwash Behind Wings," *Journal of the Aeronautical Sciences*, Vol. 18, January 1951, pp. 21-32.

[2] Brown, C. D., "Aerodynamics of Wake Vortices," *AIAA Journal*, Vol. 11, April 1973, pp. 531-536.

[3] Mason, W. H. and Marchman, J. F., III, "Far-Field Structure of Aircraft Wake Turbulence," *Journal of Aircraft*, Vol. 10, Feb. 1973, pp. 86-92.

[4] Rossow, V. J., "On the Inviscid Rolled-Up Structure of Lift Generated Vortices," NASA, TMX-62, 224, 1973.

[5] Betz, A., "Behavior of Vortex Systems," NACA TM 713; translated from *Zeitschrift fuer Angewandte Mathematik und Mechanik*, Vol. 12, 1932, pp. 164-174.

[6] Donaldson, C. duP., "A Brief Review of the Aircraft Trailing Vortex Problem," AFOSR-TR-71-1910, May 1971, Air Force Office of Scientific Research, Washington, D.C.

[7] Moore, D. W. and Saffman, P. G., "Axial Flow in Laminar Trailing Vortices," *Proceedings of the Royal Society (London)*, Ser. A., Vol. 33A, June 1973, pp. 491-508.

[8] Jordan, P. F., "Structure of Betz Vortex Cores," *Journal of Aircraft*, Vol. 10, Nov. 1973, pp. 691-693.

Effect of Aeroacoustic Interactions on Ejector Performance

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Experimental Background

THE mass entrainment performance of a family of ejectors has been measured over a range of primary air pressures and temperatures, $0 < P_{RES} \leq 120$ psig, $60 < T_{RES} \leq 1000°F$. The ejector entrained ambient air through an efficient bellmouth inlet that led to a cylindrical mixing duct of constant diameter $D = 1.375$ in. The length, L, of the duct was varied incrementally. Primary air was injected through a long, slender, convergent nozzle whose exit plane was circular, of diameter $d = 0.2656$ in. The axes of symmetry of the duct and the nozzle were coincident, the exit plane of the nozzle and the throat of the bellmouth were also coincident.

A venturi metered the primary flow rate $\dot{m}_0$, whereas the entrained mass flow $\dot{m}_I$, was computed from measurements of ambient pressure and temperature and the pressure at the throat of the bellmouth inlet. Both devices had previously been thoroughly calibrated.[1] The mixture of primary and entrained flows exhausted to the ambient. Beginning 1.1875 in. downstream from the plane of injection, wall pressure taps were drilled at quarter inch intervals along the mixing duct. Adjacent taps were spaced at 90° intervals around the circumference of the duct.

Testing began with the longest mixing duct and proceeded to ducts of decreasing length. The performance of the longer ducts, in which mixing of the two streams was nearly complete, agreed very well with the results of a simple, one-dimensional analysis in which integral expressions of mass, momentum and energy were balanced. This can be seen in the upper curve of Fig. 1. Geometries within the interval $15.75 > L/D \geq 6.5$, roughly, produced similar results: the entrainment

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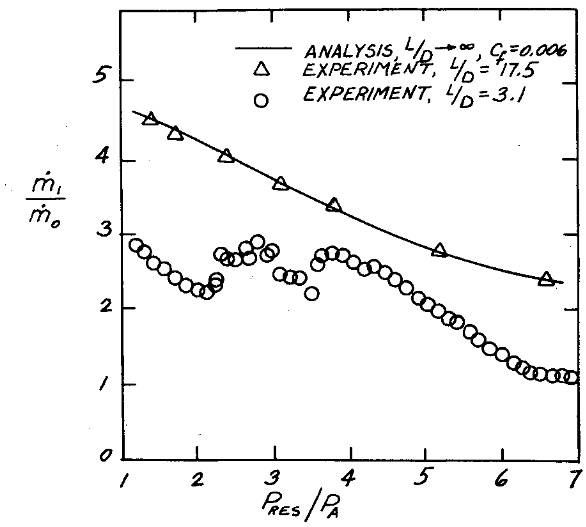

Fig. 1 Effects of reservoir pressure on mass augmentation, $T_{RES}/T_A = 1.0$.

ratio $\dot{m}_I/\dot{m}_0$ decreased with increasing reservoir pressure ratio, although overall performance dropped slightly below the fully mixed predictions as the length of the duct decreased.

The Anomaly

What seemed to be substantial scatter first appeared in the $L/D = 5.82$ data. It was not until testing was repeated at 2 psi increments of reservoir pressure that a real trend became obvious and was considerably amplified with the shorter ducts. Typical data are shown in Fig. 1. Performance at first followed the trend of the simple analysis, although at a substantial reduction that reflected incomplete mixing. A trend reversal occurred at approximately two atmospheres reservoir pressure and peaked at 2.7. A second peak at $P_{RES} = 3.7\ P_A$ brought the data near the fully mixed results. The peaks clearly indicate an accelerated mixing condition that persisted at the same pressure ratios with all of the ducts. This condition was not obvious in the mass entrainment data from the longer ducts because performance levels representative of nearly complete mixing were achieved at all pressure ratios. Nevertheless, accelerated mixing did occur and was obvious in the wall pressure data.

Figure 2 is typical and describes the increase in wall pressure, from P_{BM} at the bellmouth throat to P_A at the exhaust plane, along the length of the duct. In these tests only mixing and skin friction contribute to the pressure rise and the effect of the latter is small. The distance along the tube at which the data essentially become zero is therefore indicative of the rate of mixing. Three distinct mixing rates are obvious. At reservoir pressure ratios of three and less, the primary and entrained streams require approximately seven or eight duct diameters to complete mixing. In contrast, the $P_{RES}/P_A = 3.7$ data describe an accelerated mixing condition by achieving their terminal state much earlier, at $X/D = 5$. This is the same pressure ratio at which one of the mass entrainment peaks in Fig. 1 occurred. The pressure ratio 5 and 6.6 data suggest a retarded condition that requires nine duct diameters to mix completely.

A Proposed Mechanism

Research in the past decade has convincingly related entrainment and mixing to the large scale vortices convected along the outer edges of jets and wakes, a structure known to persist throughout a very large Reynolds number range. It was, therefore, perplexing that the same nozzle should issue jets with three distinct turbulence structures under similar conditions. Abrupt changes in the quality of the ejector's noise under different levels of reservoir pressure suggested an

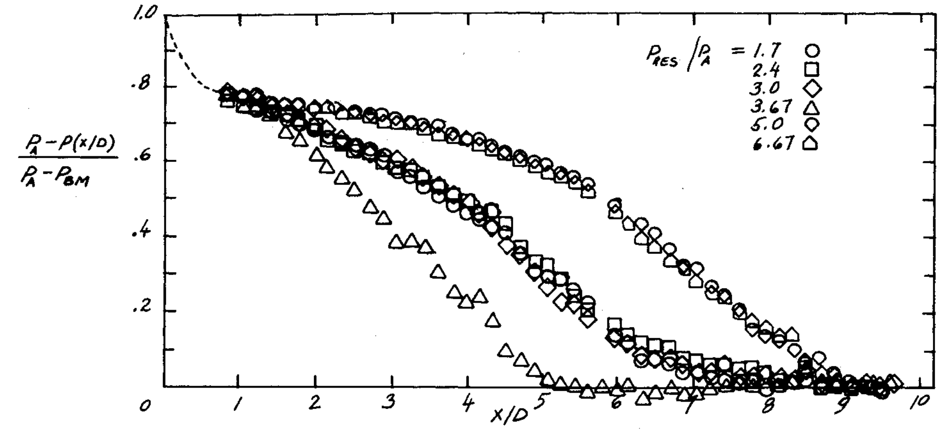

Fig. 2 Wall pressure distributions at different reservoir pressures, $T_{RES}/T_A = 1.0$. Broken line from static pressure probe.

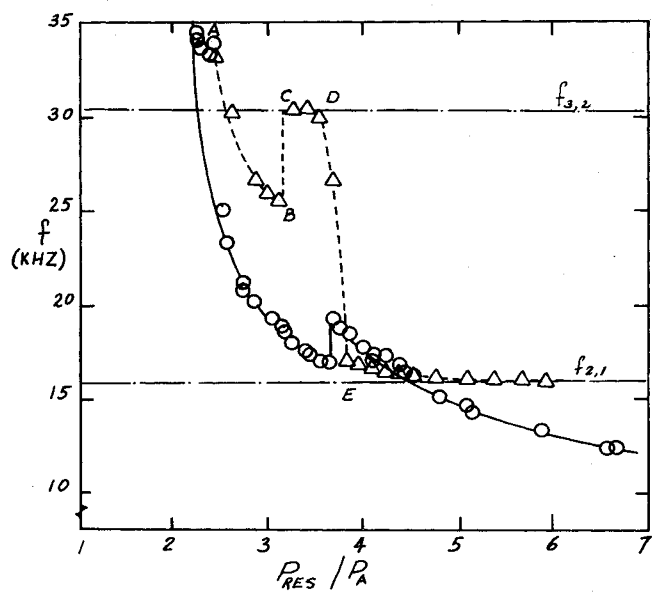

Fig. 3 Screech frequencies of free jet (o, Ref. 13) and shrouded jet (--Δ--, unreported) measured by Rosfjord and Toms.

explanation. Experience of some years ago [2, 3] had clearly identified the structural sensitivity and control of flows with large, periodic vortices to an external acoustic field. It was subsequently found that external sound influenced the spreading, decay and mixing of subsonic[4-6] and supersonic[7] jets. Powell [8] had proposed that periodic eddies in the shear layers of jets from choked nozzles generate loud screech tones as they traverse the shock cells. Or, from another point of view, the screech tones manifest the existence of periodic, large scale mixing vortices. Other works [9-12] confirm the presence of screech in ejectors. It would seem then, that the supercritical performance of compact ejectors could be influenced by an external acoustic field capable of shifting screech frequencies and altering mixing rates. Correlation of acoustic properties with the performance of an ejector appears not to have been discussed previously.

Prompted by the above, Rosfjord and Toms[13] began an investigation of the acoustic properties of the apparatus used in the present investigation. Part of their results has been presented again in Fig. 3, where the solid line and circles delineate the screech frequency of the unshrouded nozzle as a function of the reservoir pressure ratio. Also shown (triangles and broken line) are the screech tones radiated by the nozzle when positioned within the ejector. The two curves differ substantially. That the ejector frequencies are higher than the free jet frequencies up to $P_{RES}/P_A \approx 3.2$ probably relates to the thinner shear layers occasioned by the coflowing entrained stream in the former case. Thereafter, the ejector's frequency appears to assume either of two values, 31 or 16 KHZ, approximately. Under these conditions, the vortices proposed by Powell are under the control of the same acoustic field that controlled the eddies in the resonant cavity[2] referenced earlier, namely, transverse standing waves. The present duct modes follow from solutions of LaPlace's equation in cylindrical coordinates. These are Bessel functions $J_n(\alpha r)$, where n is the order of the function, α is the wave number and r is the radial ordinate. The associated eigenfrequencies are $f_{n,m}$ where m is the rank of the root of $J_n(\alpha D/2)$. The frequencies $f_{3,2}$ = 31 kHz and $f_{2,1}$ = 16 kHz have been identified in Fig. 3. Why acoustic control occurs at these frequencies and not at the many other eigenfrequencies within the same range must relate to the dynamics of the shear layer. In this regard, we note that $f_{3,2}$ and $f_{2,1}$ both derive from rotating modes and that Westley and Woolley[14] have observed disturbances spiraling around underexpanded jets.

It is interesting to interpret Fig. 1 in terms of Fig. 3. At subcritical pressure ratios, mass augmentation performance $\dot{m}_I/\dot{m}_0$, decreases as expected with P_{RES}/P_A. The turbulence structure is "natural" but eventually changes as P_{RES}/P_A approaches 2.7, where Fig. 3 suggests that the frequency of the large eddies tune to the $f_{3,2}$ standing wave frequency of the duct. The resonance apparently inhibits the turbulent energy cascade, intensifies the large vortices and accelerates the mixing process. Increasing the pressure ratio further decreases the frequency along the path A-B, Fig. 3, thereby detuning the jet and re-establishing the energy cascade.Entrainment decreases and so, too, does the performance of the ejector.

At point B, around $P_{RES}/P_A = 3$, the frequency abruptly jumps back to $f_{3,2}$ in a way that suggests a second and distinct mode of vortex generation in the jet. This mode is also in resonance with the duct and is identified by the triangles in Fig. 2. For the same reasons mentioned above, mixing is again rapid, although in this structural mode it is more rapid, and performance at $P_{RES}/P_A = 3.7$, relative to a fully mixed conditions, exceeds that at $P_{RES}/P_A = 2.7$.

Still a third mode of mixing occurs at point E, just below a pressure ration of 4. This structural mode is identified by the upper curve in Fig. 2. Resonance occurs with the $f_{2,1}$ mode of the duct. Energy must cascade quickly to the smaller vortices in this mode of turbulence because the mixing rate decreases considerably.

Conclusions

There is little doubt that the performance of short or compact ejectors could be made to rival that of longer configurations if careful attention were paid to aeroacoustic interactions. This could be essential for the success of V/STOL aircraft applications where the location of the thrust augmenting ejectors on the wings necessitates compact geometries. The experiments described in this Note were not designed to study the acoustic interactions which were, in fact, observed by chance. The previous comments are therefore speculative in nature and have been offered in the interest of stimulating research.

References

[1] Quinn, B. and Toms, H.L., "Calibration of a High Temperature and Pressure Ejector Facility with Back Pressure Effects," ARL 75-0226, June 1974, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio.

[2] Quinn, B., "Flow in the Orifice of a Resonant Cavity," *AIAA Student Journal*, Vol. 1, 1962, pp 1-6.

[3] Quinn, B., "Decomposition Tourbillionnaire d'un Sillage Laminaire," Doctoral Dissertation, Faculte des Sciences, University of Paris, May 1964, Paris, France.

[4] Vlasov, Ye. V and Ginevskiy, A.S., "Acoustic Effect on Aerodynamic Characteristics of a Turbulent Jet," FTD-MT-24-232-68, Aug. 1968, Foreign Technology Division, Wright-Patterson AFB, Ohio.

[5] Lavrov, N.V., Rosenfeld, E.I., Zhebrak, Y.A., and Smolenskiy, V.G., "Acoustical Method for Intensification of Processes of Mixing and Combustion of Natural Gas," FTD-HT-23-36-71, Feb. 1971, Foreign Technology Division, Wright-Patterson AFB, Ohio.

[6] Simcox, C.D. and Hoglund, R.F., "Acoustic Interactions with Turbulent Jets," ASME Paper No. 70-Flcs-6, June 1970.

[7] Glass, D.R., "Effects of Acoustic Feedback on the Spread and Decay of Supersonic Jets," *AIAA Journal*, Vol. 6, Oct. 1968, pp. 1890-1897.

[8] Powell, A., "On the Mechanism of Choked Jet Noise," *Proceedings of the Physics Society*, Vol. B66, 1953, pp 1039-1056.

[9] Middleton, D. and Richards, E.J., "Discrete Frequency Noise from Ejectors," *Proceedings of the Fourth International Congress on Acoustics*, Paper No. L21, Aug. 1962.

[10] Ali, M.R.M. and Anderson, J.S., "Noise Generated by Air Flow Though Circular Nozzles of Different Exit Areas," *Proceedings of the Seventh International Congress on Acoustics*, Paper No. 25N14, 1971, pp. 537-540.

[11] Mamin, V., Pykhov, F. and Rimski-Korsakov, A., "Discrete Tone Radiation Arising from a Supersonic Jet Flowing into an Unlimited Gaseous Medium and into a Cylindrical Ejector," *Proceedings of the Seventh International Congress on Acoustics*, Paper No. 24N11, 1971, pp. 469-472.

[12] Barna, P.S., "Performance and Noise Generation Studies of Supersonic Air Ejectors," *Proceedings of the Seventh International Congress on Acoustics,* Paper No. 24N15, 1971, pp. 481-483.

[13] Rosfjord, T.J. and Toms, H.L., "Recent Observations Including Temperature Dependence on Axisymmetric Jet Screech," *AIAA Journal*, Vol. 13, Oct. 1975, pp. 1394-1396.

[14] Westley, R. and Woolley, J.H., "An Investigation of the Near Noise Fields of a Choked Aisymmetric Air Jet," *Proceedings of the AFOSR-UTIAS Symposium on Aerodynamic Noise,* University of Toronto, Press, 1968, pp. 147-167.

Errata

A Finite-Element Method for Calculating Aerodynamic Coefficients of a Subsonic Airplane

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THE sentence preceding Eq. (12) should read: For an axisymmetric body with angle of attack α and yaw angle β, the pressure coefficient at surface (r,θ,x) by neglecting high-order term $(u_x/V_\infty)^2$, is

$$C_p = -\frac{2u_x \cos\alpha \cos\beta}{V_\infty} + \frac{2\sin\alpha\cos\beta}{V_\infty \cos\theta_0} \times [u_r \cos(\theta-\theta_0) - u_\theta \sin(\theta-\theta_0)] - \frac{u_r^2+u_\theta^2}{V_\infty^2} \tag{12}$$

Both the denominators of Eqs. (A4) and (A9) should read 2π instead of 4π.

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